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DETERMINATION OF INTER-LAMINAR SHEARING STRESSES USING A SUGGESTED ANALYTICAL SOLUTION IN THE COMPOSITE LAMINATED PLATES

MUHSIN J. JWEEG[1] & MUHANNAD AL-WAILY[2]

[1]Department of Mechanical Engineering, College of Engineering, Alnahrain University, Baghdad, Iraq

[2]Department of Mechanical Engineering, Faculty of Engineering, Al-Kufa University, Najaf, Iraq

ABSTRACT

In this research, it is suggested an analytical study to determine the inter-laminar shearing stress between every two layers of the laminated composite plates, types; symmetrical and un-symmetrical, cross ply and angle ply laminated plates.

To determine the value of inter-laminated shearing stress, firstly: it must find the stress in every layer of the laminated plates by determination the displacements for the plates. So that, it is suggested a solution for the laminated plates to solve the equation of motion for the composite plates by using the First-Order Shear Deformation Theory (FSDT). Also the theory of Navier solutions is used to find the behavior of the plates in two dimensions. Then, by using model analysis method, the equation of motion for the composite plate is solved to determine the values of the displacements as a function of time due to the effect of a dynamical load. As a result, the stress in every layer of the plate layers is determined and then, the inter-laminated shearing stress is found.

The results which obtained are: the frequency, response, and the stress in every layer of the plate layers. Also, it is obtained the inter-laminated shearing stress under effect of the dynamical loads by the effect of plate side-to-thickness ratio, aspect ratio, material orthotropy, and lamination scheme, number of layer of laminated plate. Finally, variable boundary conditions for the plates are studied.

The results of the displacements are compared with those found numerically (FEM) by ANSYS program. It is found there is a good agreement between the analytical and numerical results. In addition, the results are compared with another results for other research.

KEYWORDS: Analysis Plates, Dynamic Plates, Composite Laminated Plats, Inter-Laminar Shear Stresses

INTRODUCTION

The inter-laminar shear stiffness relative to the in-plane properties of a fiber reinforced composite is typically much smaller than for homogeneous materials. The reduced stiffness means that transverse shear effects are significant for much larger width-to-thickness ratios than for metal plates.

M. Witt and K. Sobczyk [6], Presented the response of laminated plates in cylindrical bending to random dynamic loading. The formulate for correlation function (and variance) of vertical displacement of a plate are obtained and the numerical results are provided for the assumed form of the correlation function of a random loading. The effect of the fiber orientation and the correlation parameter of the external loading on the mean-square value of vertical displacement are shown graphically. The results are compared with those obtained using the classical Kirchhoff and Mindlin theory for homogeneous plates.

A. K. Nayak et al. [2], Presented the deals with the transient response of initially stressed composite sandwich plates under time dependent sine, triangular and explosive blast loading. Formulations for a family of assumed strain finite element formulations are developed on the basis of the first-order shear deformation theory with a priori shear correction factors. The developed finite elements contain three displacements and two rotations of the normals about the plate's mid-plane. New results are presented to examine the effects of alternative loadings, boundary conditions and plate geometries.

ZaferKazanc [14], Presented the dynamic response of orthotropic sandwich composite plates impacted by time-dependent external blast pulses is studied by use of numerical techniques. The theory is based on classical sandwich plate theory including the large deformation effects, such as geometric non-linearities, in-plane stiffness and inertias, and shear deformation. The finite difference method is applied to solve the system of coupled non-linear equations. The results of theoretical analysis are obtained and compared with ANSYS results. Effects of the face sheet number, as well as those related to the ply-thickness, core thickness, geometrical non-linearities, and of the aspect ratio are investigated. Detailed analyses of the influence of different type of pressure pulses on dynamic response are carried out.

W. L. Yin [12], Presented a variational method involving Lekhnitskii's stress functions is used to determine the inter-laminar stresses in a multilayered strip of laminate subjected to arbitrary combinations of axial extension, bending, and twisting loads. The stress functions in each layer are approximated by polynomial functions of the thickness coordinate. The equilibrium equations, the traction-free boundary conditions, and the continuity conditions of the inter-laminar stresses are exactly satisfied in the present analysis, while the compatibility equations and the continuity of the displacements across the interfaces are enforced in an averaged sense by applying the principle of complementary virtual work.

W. L. Yin [13], also examined, the eigenvalue problem associated with the determination of the inter-laminar stresses in a laminated strip, and physical interpretations are given to the (constant) particular solutions and the complementary solutions of the problem. The case of symmetric laminates is considered in detail, and variational solutions are computed for four-layer, symmetric, cross-ply, and angle-ply laminates subjected to the three fundamental types of strain loads. Solutions based on two sets of stress functions with polynomial expansions of different degrees are compared with each other and with existing solutions to assess the accuracy. The interface values of the stress functions and their derivatives are identified as the resultant peeling and shearing forces over end intervals of the interface.

THE SUGGESTED ANALYTICAL SOLUTION

Equivalent Single-Layer Theories (ESL)

In the “ESL” theories, the displacements or stresses are expanded as a linear combination of the thickness coordinate and undetermined functions of position in the reference surface.

$$\emptyset_i(x,y,z) = \sum_{j=0}^{N_i} \emptyset_j^i(x,y).Z^j \qquad for\ \ i = 1,2,3 \tag{1}$$

Where, $N_i$are the number of terms in the expansion. ϕ_{ij} can be either displacements or stresses.

- **Classical Laminated Plates Theory (CLPT)**

The displacement filed of laminated plates are, J. S. Rao [5],

$$u_1(x,y,z,t) = u(x,y,t) - Z\frac{\partial w}{\partial x}, u_2(x,y,z,t) = v(x,y,t) - Z\frac{\partial w}{\partial y}, u_3(x,y,z,t) = w(x,y,t) \tag{2}$$

Where (u,v,w) are the displacements, along the coordinate lines, of a material point on the xy-plane.

The equations of motion are,

$$N_{x,x} + N_{xy,y} = I_1.u_{,tt} - I_2.W_{,xtt}$$
$$N_{xy,x} + N_{y,y} = I_1.v_{,tt} - I_2.W_{,ytt}$$
$$M_{x,xx} + 2.M_{xy,xy} + M_{y,yy} + q(x,y) = I_1.W_{,tt} + I_2.u_{,xtt} + I_2.v_{,ytt} - I_3.W_{,xxtt} - I_3.W_{,yytt} \quad (3)$$

Where, $I_1 = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \rho^{(k)}\, dz$ (4)

$\rho^{(K)}$ being the material density of K^{th} layer.

The laminate constitutive equations can be expressed in the form,

$$\begin{bmatrix} N_i \\ M_i \end{bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{bmatrix} \begin{bmatrix} [\varepsilon] \\ [K] \end{bmatrix} \quad (5)$$

Where,

$\varepsilon_x=u,_x$, $\varepsilon_y=v,_y$, $\gamma_{xy}=u,_y+v,_x$, $K_x=w,_{xx}$, $K_y=-w,_{yy}$, $K_{xy}=-2\, w,_{xy}$ (6)

The A_{ij}, B_{ij}, D_{ij} (i,j =1,2,6) are the respective inplane, bending –inplane coupling, and bending or twisting, respectively,

$$(A,B,D) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \overline{Q}^{(k)} (1, Z, Z^2)\, dz \quad (7)$$

Here Z_m denotes the distance from the mid-plane to the lower surface of the K^{th} layer.

Equations (3) and (5) can be conveniently expressed in the operator form as,

$$[L][\Delta] + [f] = [M][\ddot{\Delta}] \quad (8)$$

Where, $M_{11}=I_1$, $M_{12}=0$, $M_{13}=-I_2\, d_x$, $M_{22}=I_1$,$M_{23}=-I_2 d_y$, $M_{33}=I_1 - I_3\, (d_{xx}+d_{yy})$.

$[\Delta]=[u \quad v \quad w]^T$,$[f]=[0 \quad 0 \quad q(x,y,t)]^T$.

$L_{11}=A_{11}\, d_{xx} +2\, A_{16}d_{xy} +A_{66}d_{yy}$, $L_{12}=A_{12}d_{xy} +A_{16}\, d_{xx} +A_{26}d_{yy} +A_{66}d_{xy}$,

$L_{13}=-B_{11}\, d_{xxx} -B_{12}d_{xyy} -3\, B_{16}d_{xxy}- B_{26}d_{yyy} -2\, B_{66}d_{xyy}$,

$L_{22}=2A_{26}d_{xy} +A_{66}\, d_{xx} +A_{22}d_{yy}$,

$L_{23}=-B_{16}\, d_{xxx} -3\, B_{26}d_{xyy} -2B_{66}d_{xxy} -B_{12}d_{xxy} -B_{22}d_{yyy}$,

$L_{33}= -D_{11}d_{xxxx} -2\, D_{12}d_{xxyy} -4\, D_{16}d_{xxxy} -4\, D_{26}d_{xyyy} -4\, D_{66}d_{xxyy} -D_{22}d_{yyyy}$. (9)

- **First-Order Shear Deformation Theory (FSDT)**

This theory accounts for linear variation of in-plane displacements through the thickness,

$u_1(x,y,z,t)=u(x,y,t)+Z\, \psi_x(x,y,t)$, $u_2(x,y,z,t)=v(x,y,t)+Z\, \psi_y(x,y,t)$, $u_3(x,y,z,t)=w(x,y,t)$. (10)

Where,

t is the time; u_1 , u_2 , u_3 are the displacements in x,y,z directions, respectively; and ψ_x and ψ_y are the slopes in the xy and yz planes due to bending only.

The equations of motion are:

$N_{x,x}+N_{xy,y}=I_1u_{,tt}+I_2\psi_{x,tt}$

$N_{xy,x}+N_{y,y}=I_1v_{,tt}+I_2\psi_{y,tt}$

$N_{xz,x}+N_{yz,y}+q(x,y,t)=I_1w_{,tt}$

$M_{x,x}+M_{xy,y}-N_{xz}=I_2u_{,tt}+I_3\psi_{x,tt}$

$M_{xy,x}+M_{y,y}-N_{yz}=I_2v_{,tt}+I_3\psi_{y,tt}$ (11)

Where,

$$(I_1,I_2,I_3)=\sum_{k=1}^{n}\int_{z_{k-1}}^{z_k}\rho^{(k)}(1,Z,Z^2)\,dz) \quad (12)$$

The laminated constitutive equations can be expressed in the form,

$$\begin{bmatrix} N \\ M \end{bmatrix}=\begin{bmatrix} A & B \\ B & D \end{bmatrix}\begin{bmatrix} \varepsilon \\ K \end{bmatrix},\begin{bmatrix} N_{yz} \\ N_{xz} \end{bmatrix}=\begin{bmatrix} k_{44}^2A_{44} & k_{45}^2A_{45} \\ k_{45}^2A_{45} & k_{55}^2A_{55} \end{bmatrix}\begin{bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{bmatrix} \quad (13)$$

Where,

$\varepsilon_x=u_{,x}$, $\varepsilon_y=v_{,y}$, $\gamma_{xy}=u_{,y}+v_{,x}$, $\gamma_{yz}=\psi_y+w_{,y}$, $\gamma_{xz}=\psi_x+w_{,x}$,$K_x=\psi_{x,x}$, $K_y=\psi_{y,y}$, $K_{xy}=\psi_{x,y}+\psi_{y,x}$ (14)

K_{45}, K_{44} and K_{55} are correction factors.

Equations (11) and (13) can be conveniently expressed in the operator form as,

$$[L][\Delta]+[f]=[M][\ddot{\Delta}] \quad (15)$$

Where, $[\Delta]=[u \quad v \quad w \quad \psi x \quad \psi y]^T$, $[F]=[0 \quad 0 \quad q(x,y,t)]^T$

And, $M_{11}=M_{22}=M_{33}=I_1$, $M_{44}=M_{55}=I_3$, $M_{14}=M_{25}=I_2$, and other terms of $M_{ij}=0$ (for $i\neq j$).

And, $L_{11}=A_{11}d_{xx}+2A_{16}d_{xy}+A_{66}d_{yy}$, $L_{12}=A_{12}d_{xy}+A_{16}d_{xx}+A_{26}d_{yy}+A_{66}d_{xy}$, $L_{13}=0$,

$L_{14}=B_{11}d_{xx}+2B_{16}d_{xy}+B_{66}d_{yy}$, $L_{15}=B_{12}d_{xy}+B_{16}d_{xx}+B_{26}d_{yy}+B_{66}d_{xy}$

$L_{22}=2A_{26}d_{xy}+A_{66}d_{xx}+A_{22}d_{yy}$, $L_{23}=0$, $L_{24}=B_{16}d_{xx}+B_{66}d_{xy}+B_{12}d_{xy}+B_{26}d_{yy}$,

$L_{25}=2B_{26}d_{xy}+B_{66}d_{xx}+B_{22}d_{yy}$, $L_{33}=2A_{45}d_{xy}+A_{55}d_{xx}$, $L_{34}=A_{55}d_x+A_{45}d_y$,

$L_{35}=A_{45}d_x+A_{44}d_y$, $L_{44}=D_{11}d_{xx}+2D_{16}d_{xy}+D_{66}d_{yy}-A_{55}$,

$L_{45}=D_{12}d_{xy}+D_{16}d_{xx}+D_{26}d_{yy}+D_{66}d_{xy}-A_{45}$, $L_{55}=2D_{26}d_{xy}+D_{66}d_{xx}+D_{22}d_{yy}-A_{44}$. (16)

Actual Displacements for Simply-Supported Laminated Plate

- **Cross-Ply Laminated Plate**

The general actual displacements for cross-ply laminated plate are, Bose and Reddy [3],

$$u(x,y,t)=\cos\alpha x.\sin\beta y.u(t)=\bar{u}(x,y).u(t)$$
$$v(x,y,t)=\sin\alpha x.\cos\beta y.v(t)=\bar{v}(x,y).v(t)$$
$$w(x,y,t)=\sin\alpha x.\sin\beta y.w(t)=\bar{w}(x,y).w(t)$$
$$\psi_x(x,y,t)=\cos\alpha x.\sin\beta y.\psi_x=\bar{\psi}_x(x,y).\psi(t)$$
$$\psi_y(x,y,t)=\sin\alpha x.\cos\beta y.\psi_y(t)=\bar{\psi}_y(x,y).\psi_y(t) \qquad (17)$$

- **Angle-Ply Laminated Plate**

The general actual displacements for Angle-ply laminated plate are,

$$u(x,y,t)=\sin\alpha x.\cos\beta y.u(t)=\bar{u}(x,y).u(t)$$
$$v(x,y,t)=\cos\alpha x.\sin\beta y.v(t)=\bar{v}(x,y).v(t)$$
$$w(x,y,t)=\sin\alpha x.\sin\beta y.w(t)=\bar{w}(x,y).w(t)$$
$$\psi_x(x,y,t)=\cos\alpha x.\sin\beta y.\psi_x=\bar{\psi}_x(x,y).\psi(t)$$
$$\psi_y(x,y,t)=\sin\alpha x.\cos\beta y.\psi_y(t)=\bar{\psi}_y(x,y).\psi_y(t) \qquad (18)$$

General Solution for Equations of Motion

The general equations of motion are,

$$[L][\Delta]+[f]=[M][\ddot{\Delta}] \qquad (19)$$

By substituting the actual displacements, equation (17) or (18), into equation (19), the by premultiplying the result by [$\bar{\Delta}$ (x,y)]^T and integral of xy, gives,

$$[M][\ddot{\Delta}]+[K][\Delta]=[F] \qquad (20)$$

Where, $[\bar{\Delta}(x,y)]=[\bar{u}(x,y) \quad \bar{v}(x,y) \quad ....]^T$ (21)

And [M] and [K] are mass and stiffness matrices, respectively; [Δ(t)] and [F] are displacement of time and load vector, respectively.

Cross-Ply Laminated Plate

By using, depended on theory using, equation (17) into equation (19), gives,

$$[M][\ddot{\Delta}]+[K][\Delta]=[F] \qquad (22)$$

Where, [M], [K], [Δ(t)] and [F] as:

- **CLPT**

$K_{11}=\alpha^2A_{11}+\beta^2A_{66}$, $K_{12}=\alpha\beta(A_{12}+A_{66})$, $K_{13}=-\alpha^3B_{11}$, $K_{22}=\alpha^2A_{66}+\beta^2A_{22}$, $K_{23}=-\beta^3B_{22}$

$K_{33}=\alpha^4D_{11}+\alpha^2\beta^2(2D_{12}+4D_{66})+\beta^4D_{22}$. $M_{11}=M_{22}=M_{33}=I_1$, $M_{ij}=0$ for $i\neq j$.

And, $[\Delta(t)]=[u(t) \;\; v(t) \;\; w(t)]^T$, $F(t)=[0 \quad 0 \quad \bar{q}(t)]^T$

Where, $\bar{q}(t)=\frac{4}{ab}\int_0^b\int_0^a \sin\alpha x.\sin\beta y.q(x,y).dx.dy.f(t)$

- **FSDT**

$K_{11}=\alpha^2A_{11}+\beta^2A_{66}$, $K_{12}=\alpha\beta\ (A_{12}+A_{66})$, $K_{13}=0$, $K_{14}=\alpha^2B_{11}$, $K_{15}=0$,

$K_{22}=\alpha^2A_{66}+\beta^2A_{22}$, $K_{23}=0$, $K_{24}=0$, $K_{25}=\beta^2B_{22}$, $K_{33}=\alpha^2A_{55}+\beta^2A_{44}$, $K_{34}=\alpha A_{55}$,

$K_{35}=\beta A_{44}$, $K_{44}=\alpha^2D_{11}+\beta^2D_{66}+A_{55}$, $K_{45}=\alpha\beta(D_{12}+D_{66})$, $K_{55}=\alpha^2D_{66}+\beta^2D_{22}+A_{44}$.

And, [M] as in equation (16), $[\Delta(t)]=[u(t)\quad v(t)\quad w(t)\quad \psi x(t)\quad \psi y(t)]^T$, $F(t)=\begin{bmatrix}0 & 0 & \bar{q}(t) & 0 & 0\end{bmatrix}^T$.

The [M] and [K] matrix for symmetric cross-ply are as for anti symmetric cross-ply for subjected ($B_{ij}= E_{ij}=G_{ij}=0$).

Angle-Ply Laminated Plate

By suing, depended at theory using, equation (18) in to equation (19), gives,

$$[M][\ddot{\Delta}]+[K][\Delta]=[F] \tag{23}$$

Where, [M], [K], [Δ(t)] and [F] as:

- **CLPT**

$K_{11}=\alpha^2A_{11}+\beta^2A_{16}$, $K_{12}=\alpha\beta(A_{12}+A_{66})$, $K_{13}=-3\alpha^2\beta B_{16}-\beta^3B_{26}$, $K_{22}=\alpha^2A_{66}+\beta^2A^{22}$,

$K_{23}=-\alpha^3B_{16}-3\alpha\beta^2B_{26}$, $K_{33}=\alpha^4D_{11}+2\alpha^2\beta^2D_{12}+4\alpha^2\beta^2D_{66}+\beta^4D_{22}$.

And, $M_{11}=M_{22}=M_{33}=I_1$, $M_{ij}=0$ for $i\neq j$; $[\Delta(t)]=[u(t)\quad v(t)\quad w(t)]^T$; $[F]=\begin{bmatrix}0 & 0 & \bar{q}(t)\end{bmatrix}^T$.

- **FSDT**

$K_{11}=\alpha^2A_{11}+\beta^2A_{16}$, $K_{12}=\alpha\beta(A_{12}+A_{66})$, $K_{13}=0$, $K_{14}=2\alpha\beta B_{16}$, $K_{15}=\alpha^2B_{16}+\beta^2B_{26}$

$K_{22}=\alpha^2A_{66}+\beta^2A_{22}$, $K_{23}=0$, $K_{24}=\alpha^2B_{16}+\beta^2B_{26}$, $K_{25}=2\alpha\beta B_{26}$, $K_{33}=\alpha^2A_{55}+\beta^2A_{44}$,

$K_{34}=\alpha A_{55}$, $K_{35}=\beta A_{44}$, $K_{44}=\alpha^2D_{11}+\beta^2D_{66}+A_{55}$, $K_{45}=\alpha\beta(D_{12}+D_{66})$,

$K_{55}=\alpha^2D_{66}+\beta^2D_{22}+A_{44}$. And, [F] and [Δ(t)] as in equation (40).

And, $M_{11}=M_{22}=M_{33}=I_1$, $M_{44}=M_{55}=I_3$, $M_{ij}=0$ for $i\neq j$.

When external forces act on a multi-degree of freedom system undergoes forced vibration. For a system with (n) coordinates or degrees of freedom, the governing equations of motion are a set of (n) coupled ordinary differential equations of second order. The solution of these equations becomes more complex when the degree of freedom of the system (n) is large and/or when the forcing functions are non-periodic. In such cases, a more convenient method known as "Modal analysis" can be used to solve the problem, **SingiresuS. Rao [11]**.

Inter-Laminar Shear Stresses

Inter-laminar stresses are one of the failure mechanisms uniquely characteristic of composite materials. The free body diagram of each layer of a laminate shown in **Figure 1**, Robert M. Jones [9], study useful in understanding the physical mechanism of shear transfer between layers.

Then, the fact that τ_{xz} must be zero on a free edge means that the couple caused by τ_{xz} acting along the other edges of the free body must be reacted. The only possible reacting couple to satisfy moment equilibrium is caused by τ_{xz} acting on part of the lower face of the layers at the interface with the next layer.

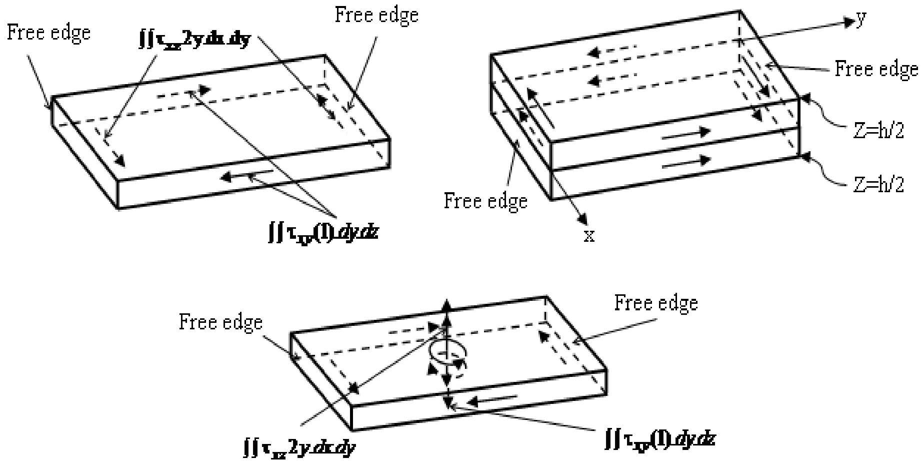

Figure 1: Inter-Laminar Shear Stress Mechanism

In the laminated plates, no account is taken of inter-laminar stresses such as σ_z ,τ_{xz} , and τ_{yz} which are shown in **Figure 2**, **Robert M. Jones [9]**.

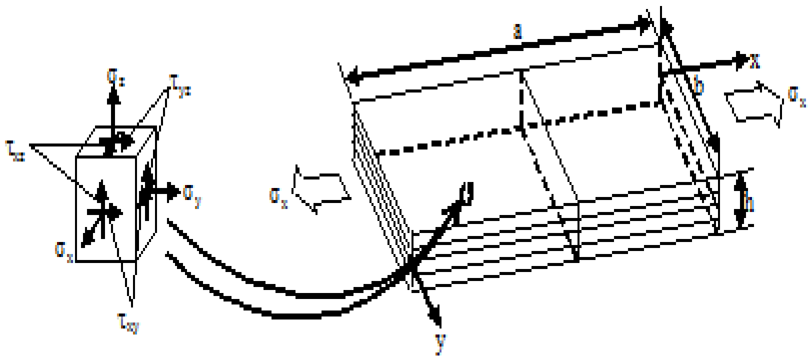

Figure 2: Laminate Geometry and Stresses

The inter-laminar shear stresses are determined from the first two equations of equilibrium, **J. S. Rao [5]**,

$$\begin{aligned}\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}+\frac{\partial\tau_{xz}}{\partial z}&=\rho\frac{\partial^2 u_1}{\partial t^2}\\ \frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\sigma_y}{\partial y}+\frac{\partial\tau_{yz}}{\partial z}&=\rho\frac{\partial^2 u_2}{\partial t^2}\end{aligned} \tag{24}$$

To solve equation (24), the stresses should be evaluated depending upon the determined of displacements by using dynamic analysis.

Integrating equations (24) with respect to z and using equation of the stresses in kth layer of (CLPT), M. Al_Waily [7], gives,

$$\begin{aligned}\begin{bmatrix}\tau_{xz}\\ \tau_{yz}\end{bmatrix}_j&=\begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16}\\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66}\end{bmatrix}_j\left[-Z\begin{bmatrix}u_{,xx}\\ v_{,xy}\\ u_{,xy}+v_{,xx}\end{bmatrix}+\frac{Z^2}{2}\begin{bmatrix}w_{,xxx}\\ w_{,xyy}\\ 2w_{,xxy}\end{bmatrix}\right]\\ &+\begin{bmatrix}\overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66}\\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26}\end{bmatrix}_j\left[-Z\begin{bmatrix}u_{,xy}\\ v_{,yy}\\ u_{,yy}+v_{,xy}\end{bmatrix}+\frac{Z^2}{2}\begin{bmatrix}w_{,xxy}\\ w_{,yyy}\\ 2w_{,xyy}\end{bmatrix}\right]+\rho\left[Z\begin{bmatrix}u_{,tt}\\ v_{,tt}\end{bmatrix}-\frac{Z^2}{2}\begin{bmatrix}w_{,xtt}\\ w_{,ytt}\end{bmatrix}\right]+\begin{bmatrix}f(x,y)\\ g(x,y)\end{bmatrix}\end{aligned} \tag{25}$$

Where u(x,y,t), v(x,y,t), w(x,y,t) are evaluated by dynamic analysis of composite plate for (CLPT). And f(x,y), g(x,y) are the functions of integration to be determined from the inter laminar continuity conditions for the intermediate layers and-zero shear traction condition on the top and bottom surfaces.

Integral the equations (24) with respect to z and using equation of the stresses in k^{th} layer of (FSDT), **M. Al_Waily [7],** gives,

$$\begin{bmatrix}\tau_{xz}\\ \tau_{yz}\end{bmatrix}_j = \begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16}\\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66}\end{bmatrix}_j \left[-Z\begin{bmatrix}u_{,xx}\\ v_{,xy}\\ u_{,xy}+v_{,xx}\end{bmatrix} - \frac{Z^2}{2}\begin{bmatrix}\psi_{x,xx}\\ \psi_{y,xy}\\ \psi_{x,xy}+\psi_{y,xx}\end{bmatrix}\right] + \tag{26}$$

$$\begin{bmatrix}\overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66}\\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26}\end{bmatrix}_j \left[-Z\begin{bmatrix}u_{,xy}\\ v_{,yy}\\ u_{,yy}+v_{,xy}\end{bmatrix} - \frac{Z^2}{2}\begin{bmatrix}\psi_{x,xy}\\ \psi_{y,yy}\\ \psi_{x,yy}+\psi_{y,xy}\end{bmatrix}\right] + \rho\left[Z\begin{bmatrix}u_{,tt}\\ v_{,tt}\end{bmatrix} + \frac{Z^2}{2}\begin{bmatrix}\psi_{x,tt}\\ \psi_{y,tt}\end{bmatrix}\right] + \begin{bmatrix}f(x,y)\\ g(x,y)\end{bmatrix}$$

Where, u(x,y,t), v(x,y,t), w(x,y,t), ψ_x(x,y,t), ψ_y(x,y,t) are evaluated by dynamic analysis of composite plate for (FSDT).

The computer programs designed in this work are concerned with solving the dynamic problems for displacements, stresses in each layers σ_x; σ_y; τ_{xy}, and inter laminar stresses between each two layers of composite laminated plates using any theory for laminated plates. The computer programs constructed herein are coded in "Fortran Power Station 4.0" language. The program defined the displacement in x, y, and z-direction of plates and the stresses of plates,as a faction of x, y, and time, then evaluated the inter laminar shear stresses between each two layers of laminated plates, solving by using of (FSDT).The input required of program are, the orthotropic properties of lamina (E_1, E_2, G_{12}, and ν_{12}) and, the dimensions of laminated plate (a, b, and h). And, the output of program are, displacement of laminated plate as a function of x; y; t, (u, v, w, ψ_x, and ψ_y), stresses of laminated plate in each layers as a function of x; y; z; t, (σ_x, σ_y, τ_{xy}), and inter laminar shear stress between each two layers of laminated plate as a function of x; y; z; and t, (τ_{xz}, τ_{yz}).

RESULTS AND DISCUSSIONS

The case study discussed here is a un-stiffened laminated simple supported plate **Figure 3**. with dimensions and material properties give below using the first-order shear deformation theory (FSDT) and applying the suggested analytical solution and finite element method by used of (ANSYS).

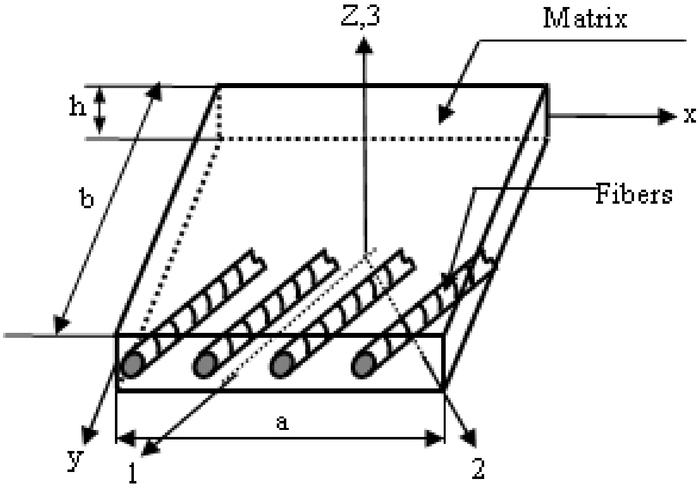

Figure 3: Dimensions and Directions of Un-Stiffened Laminated Plate

COMPARISON RESULTS

Figures 4 and 5 shows a comparison of the present work solutions by Analytical and finite elements method with the numerical solution of Reddy, J. N. [10].

Figure 4, shows a comparison of the present work solutions by Analytical and finite elements method with the numerical solution of Reddy, J. N. [10] they are given for two layer simply supported cross-ply laminated plate subjected to sinusoidal Pulse loading q(x,y,t)=P(x,y), for, $P(x,y)=q_o \sin(\pi x/a) \sin(\pi y/b)$, q_o=10 N/cm^2.

And the properties and dimensions of plate, are, $E_2=2.1*10^6$ N/cm^2, $E_1/E_2=25$, ν=0.25, $G_{12}=G_{13}=G_{23}=0.5E_2$, ρ=800 Kg/m^3, a=b=25 cm, h=5 cm.

Figure 5 shows a comparison of the present work with the numerical solution of **Reddy, J. N. [10]** they are given for simply supported two layer cross-ply laminated plate subjected to sinusoidal Pulse loading q(x,y,t)=P(x,y), for, $P(x,y)=q_o \sin(\pi x/a)\sin(\pi y/b)$,q_o=10 N/cm^2 ,

And the properties and dimensions of plate, are, $E_2=2.1*10^6$ N/cm^2, $E_1/E_2=25$, ν=0.25, $G_{12}=G_{13}=G_{23}=0.5E_2$, ρ=800 Kg/m^3, a=b=25 cm , h=1 cm.

Frequency Results

Figures 6, 7, 8, and 9 shows the natural frequency of laminated plate with properties and dimensions as, A. A. Khdeir and Reddy [1], E_1=130.8 Gpa, E_2=10.6 Gpa, ν_{12}=0.25, $G_{12}=G_{13}$=6 Gpa, G_{23}=3.4 Gpa, a=1 m, b=1 m, h=0.02 m

Figure 6.shows the effect of number of layer on the natural frequency for different aspect ratio, for a=1 m, of anti-symmetric cross-ply simply supported laminated plates. The figure shows that the natural frequency of plates increases with increasing the number of layers, and the aspect ratio. The increases of frequency are 32.2%, 3.85%, 1.265%, 0.57% for increasing layers (2 to 4), (4 to 6), (6 to8), (8 to 10) respectively of aspect ratio a/b=2 and 49.12%, 184.54% for increase aspect ratio (0.5 to 1), (1 to 2) respectively for number of layer N=4.

Figure 7.shows the effect of fiber orientation on the natural frequency for different number of layer of anti-symmetric angle-ply simply supported laminated plates. The figure shows that the natural frequency of plates increase with increasing the number of layers and the fiber orientation to optimum angle (θ=45^o). The increases of frequency with increasing the angle θ form 0o to 45o are 19.7%, 26.34% for increase layers (2 to 4), (4 to 8) respectively.

Figure 8.shows the effect of fiber orientation on the natural frequency for different modulus ratio (E_1/E_2) for four layers of anti-symmetric angle-ply simply supported laminated plates. The figure shows that the natural frequency of plates increase with increasing of the modulus ratio (E_1/E_2). The increase of frequency are 35.15%, 20.16% for increase E_1/E_2 (10 to 20), (20 to 30) respectively for θ=45^o.

Figure 9.shows the effect of fiber orientation on the natural frequency for different length-to-thickness ratio (a/h), for a=1 m, for four layers of anti-symmetric angle-ply simply supported laminated plates. The figure shows that the natural frequency of plates increases with decreasing (a/h) ratio, and increasing the thickness h of plates. The increase of frequency are 96.23%, 23.3% for decreasing a/h (50 to 25), (25 to 20) respectively for θ=45^o.

Deflection Results

Figures 10 and 11, Shows the central deflection of laminated plate with different boundary condition of plate with properties and dimensions as, A. A. Khdeir and Reddy [1], E_1=130.8 Gpa, E_2=10.6 Gpa, G_{12}=G_{13}=6 Gpa, G_{23}=3.4 Gpa, ν12=0.25, a=1 m, b=1 m, h=0.02 m.

Figure 10, shows the central deflection for different boundary conditions ,simply supported, clamped, (simply supported at (x=0,a) and clamped at (y=0,b)), (free edges at (x=0,a) and simply supported at (y=0,b)), free edges at (x=0,a) and clamped at (y=0,b), simply supported at ends of plate ((x=0,y=0), (x=a, y=0), (x=0, y=b), (x=a, y=b)), and clamped at ends of plate ((x=0,y=0), (x=a, y=0), (x=0, y=b), (x=a, y=b)) subjected to sinusoidal ramp loading, solution by (F.E.M). From the results, the maximum deflection of simply supported plate at ((x=0,y=0), (x=a, y=0), (x=0, y=b), (x=a, y=b)).

Figure 11. shows the maximum deflection for different boundary conditions simply supported at ((x=0,y=0), (x=a, y=0), (x=0, y=b), (x=a, y=b)), clamped at ends of plate ((x=0,y=0), (x=a, y=0), (x=0, y=b), (x=a, y=b)), and cant-lever plate (clamped at x=0 and free at x=a,y=0,b) subjected to sinusoidal ramp loading, solved by (F.E.M). From the results, the maximum deflection occurs for cantilever plate (clamped at x=0 and free at x=a, y=0,b).

From Figures 10 and 11, shows that the maximum deflection occurred when the laminated plate supported as cantilever plate. The following properties were used for simply supported laminated plates, in figure.12. forq_o=10 N/cm^2, t_o=0.0005 sec, simply supported laminated plates, J. N. Reddy [4], E_2=2.1*10^6 N/cm^2, E_1/E_2=25, G_{12}=G_{13}=G_{23}=0.5E_2, ρ=1500 Kg/m^3, ν=0.25, a=b=25 cm , h=5 cm.

Figure 12.represents the variation of central transverse deflection with time for angle-ply and cross-ply laminated under sinusoidal Ramp loading solution by analytical and (F.E.M). The (0/90/…) laminated higher in magnitude than the (45/-45/…) laminated because at (θ=450/-450/…) the extension and bending stiffnesses A16, A26, D16 and D26 appear to have a significant effect while at (θ=00/900/…).

The following properties were used for simply supported Laminated Plates, Figures. 13 to 17, A. A. Khdeir and Reddy [1], E_1=130.8 Gpa, E_2=10.6 Gpa, G_{13}=G_{23}=6 Gpa , G_{23}=3.4 Gpa, ρ=1580 Kg/m^3,ν=0.25, a=b=1 m, h=0.02 m, and q_o=10 kN/m^2 to=0.05 sec.

Figure 13, shows the effect of the aspect ratio (a/b) on the deflection of the simply supported anti symmetric cross-ply laminated plates (a=1 m) subjected to sinusoidal Ramp loading solution by analytical and (F.E.M). From the results, the increase of (a/b) ratio increases the deflection.

Figure 14, shows the effect of the (a/h) ratio on the deflection of the simply supported anti symmetric cross-ply laminated plates (a=1 m)subjected to sinusoidal sine loading solution by analytical and (F.E.M). From the results, the increase of (a/h) ratio increases the deflection of laminated plates.

Figure 15.shows the effect of the number of layer of simply supported anti symmetric cross-ply laminated plates on the deflection of plate subjected to sinusoidal Pulse loading solution by analytical and (F.E.M). The central deflection of laminated plates decreases with increasing number of layers.

Figure 16, shows the effect of the lamination angle ($θ^0$) on the deflection of simply supported anti symmetric angle-ply laminated plates under sinusoidal ramp loading solution by analytical and (F.E.M). It is apparent from the results that the deflection decreases with increasing the angle of laminated.

Figure 17, shows the effect of the number of layer of simply supported anti symmetric angle-ply laminated plates on the deflection of plate subjected to sinusoidal sine loading solution by analytical and (F.E.M). The central deflection of laminated plates decreases with increasing number of layers.

Stress Results

The following properties were used for simply supported laminated plates, for analytical solutions, in figures 18 to 23, q_o=10 kN/m^2, t_o=0.05 sec, A. A. Khdeir and Reddy [1], for simply supported, E_1=130.8 Gpa, E_2=10.6 Gpa,G_{12}=G_{13}=6 Gpa, G_{23}=3.4 Gpa, ρ=1580 Kg/m^3, ν=0.28, a=b=1 m, h=0.02 m.

Figure 18, represents the stress-x in each layer, at the middle of layers, with time for four layers Anti symmetric cross-ply (0/90/0/…) laminated plates under uniformly ramp loading $q(x,y,t)=q_o\, t/t_o$ for q_o=1 N/cm^2 ,t_o=0.05 sec), at x=a/2, y=b/2. The maximum value of σx is at layer-1 and the stress-x areantisymmetric about the middle plane.

Figure 19, represents the stress-x in layer-1, at the middle of layer, with time for different number of layer for Anti symmetric cross-ply (0/90/0/…) laminated plates under uniformly ramp loading $q\ (x,y,t)=q_o\ t/t_o$ for q_o=1 N/cm^2, t_o=0.05 sec, at x=a/2, y=b/2. The value of σx at layer-1 increase with increase the number of layers

Figure 20, represents the effect of the lamination angle(θ^0) on the σ_x at layer-1 for four layers anti symmetric angle-ply laminated plates under uniformly ramp loading, at x=a/2, y=b/2. From the results the σx decreases with the increase of the angle of laminated to the 45^0, the minimum value at 45^0 and the maximum value at 0^o.

Figure 21, represents the comparison of stress-x with stress-y at layer-1 for four layers anti symmetric cross-ply laminated plates for difference E_1/E_2 under uniformly pulse loading, at x=a/2, y=b/2. From the results, stresses-x are more than stresses-y at $E_1/E_2\neq1$ and Stress-x equal stress-y for E_1/E_2=1.

Figure 22. represents the comparison stress-x with stress-y at layer-1 for four layers anti symmetric cross-ply laminated plates for difference aspect ratio under uniformly ramp loading, at x=a/2, y=b/2. From the results, stresses-x are more than stresses-y.

Figure 23.represents the stress-y in layer-1, at the middle of layer, with time for different number of layer for Anti symmetric cross-ply (0/90/0/…) laminated plates under uniformly sine loading $q(x,y,t)=q_o\ \sin(\pi t/t_o)$ for q_o=1 N/cm^2, t_o=0.05 sec, at x=a/2, y=b/2. The value of σ_y at layer-1 decreases with the increase of the number of layers.

Inter-Laminar Shear Stresses Results

The case study discussed here is a laminated simple supported plate with dimensions and material properties used, **A. A. Khdeir and Reddy [1]**; using the first-order shear deformation theory (FSDT) and applying the suggested analytical solution, to evaluate the inter laminar shear stresses of laminated plates,

E_1=130.8 Gpa, E_2=10.6 Gpa, G_{12}=6 Gpa, ρ=1580 kg/m^3, ν_{12}=0.25,

Length of plate=1 m, Width of plate=1 m, Thickness of plate= 0.02 m,

Dynamic distributed load= 10 kN/m, Initial time of load= 0.05 sec

Figure 24 shows the inter laminar shear stress τ_{xz} for four layer antisymmetric cross-ply laminated plates subjected to uniformly ramp loading. The maximum shear stress τ_{xz} occurs at the middle plane (0.417 Mpa).

Figure 25 represents the effect of number of layers on the inter laminar shear stress τ_{xz} between layers(1-2) for anti symmetric cross-ply laminated plates subjected to uniform sinusoidal loading. The shear stress τ_{xz} decreases with increasing the number of layers. The decrease in τ_{xz} are 20.5% and 20% for increasing layers from (4 to 6) and (6 to 8) respectively.

Figure 26 shows the inter laminar shear stress τ_{yz} for four layer anti symmetric cross-ply laminated plates subjected to uniformly ramp loading. The maximum shear stress τ_{yz} occurs at the middle plane.

Figure 27 shows the effect of number of layer on the inter laminar shear stress τ_{yz} between layers(1-2) for anti symmetric cross-ply laminated plates subjected to uniform sinusoidal loading. The shear stress τ_{yz} decreases with increasing the number of layers. The decrease in τ_{yz} are 63.8%, 33.6%, and 22.7% for increasing layers from (2 to 4), (4 to 6), and (6 to 8) respectively.

Figure 28 represents the comparison τ_{xz} with τ_{yz} between layers for four layer anti symmetric cross-ply laminated plates subjected to uniformly ramp loading. The τ_{xz} equal to τ_{yz} at the middle plane and τ_{xz} greater than τ_{yz} with 74.4% between other layers.

Figure 29 represents the effect of aspect ratio (a=1) on shear stress τ_{xz} at middle plane for four layer anti symmetric cross-ply laminated plates subjected to uniformly pulse loading. In addition, the τ_{xz} decreases with increasing the aspect ratio. The decrease in τ_{xz} are 45.6% and 81.15% for increasing aspect ratio from (0.5 to1) and (1 to 2) respectively.

Figure 30 represents the comparison of τ_{xz} with τ_{yz} at middle plane for different aspect ratio (a=1) for four layer anti symmetric cross-ply laminated plates subjected to uniformly pulse loading. The shear τ_{xz} is greater than the τ_{yz} at (a/b=0.5) and τ_{xz} is less than τ_{yz} at (a/b=2). The τ_{yz} increases with increasing of the aspect ratio.

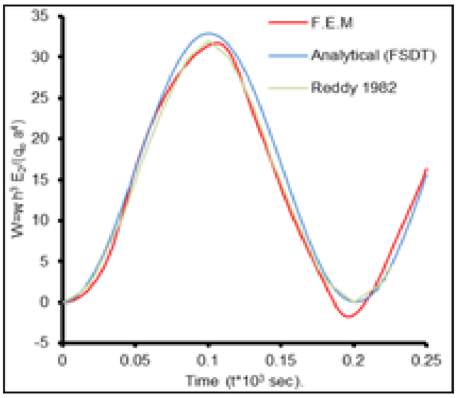

Figure 4: Central Deflection Due to Sinusoidal Pulse Loading for Two Layer

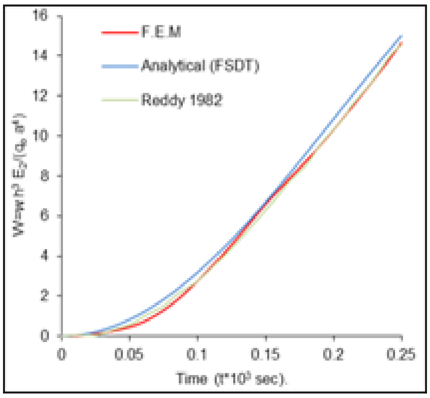

Figure 5: Central Deflection Due to Sinusoidal Pulse Loading for Two Layer

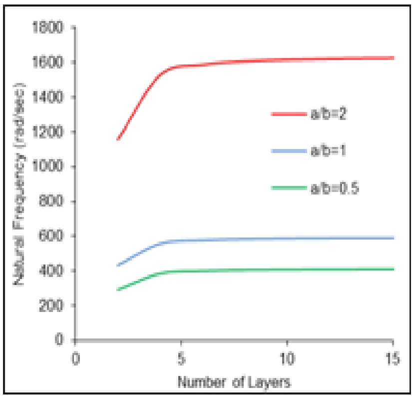

Figure 6: The Natural Frequency for Simply-Supported Anti-Symmetric Cross-Ply Plates

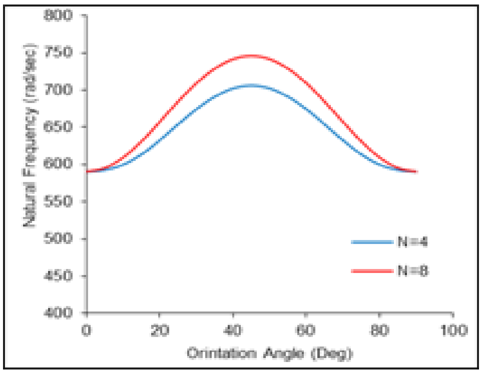

Figure 7

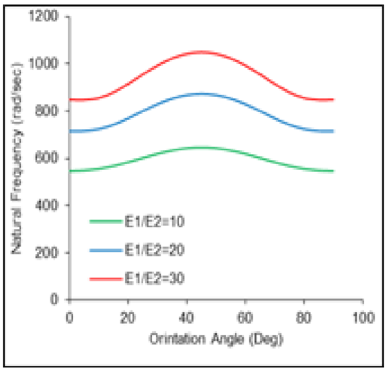

Figure 8

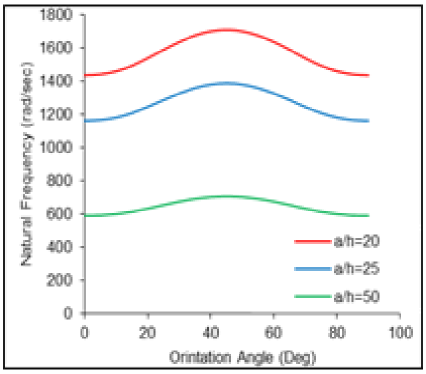

Figure 9

Figure 7, 8, 9: The Natural Frequency for Simply-Supported Anti-Symmetric Angle-Ply Laminated Plates

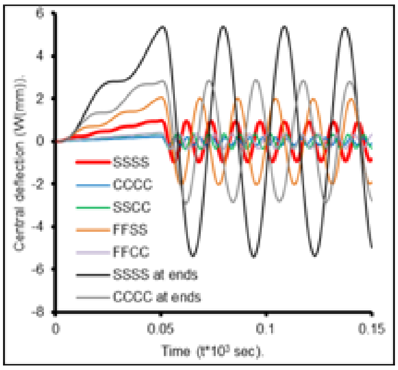

Figure 10: Central Deflection for Different Boundary Conditions for Cross-Ply Laminated Plates under Sinusoidal Ramp Loading for N=4

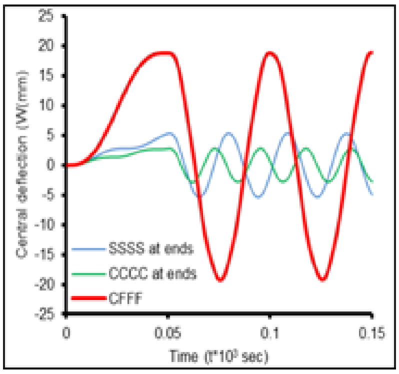

Figure 11: Maximum Deflection for Different Boundary Conditions for Cross-Ply Laminated Plates under Sinusoidal Ramp Loading for N=4

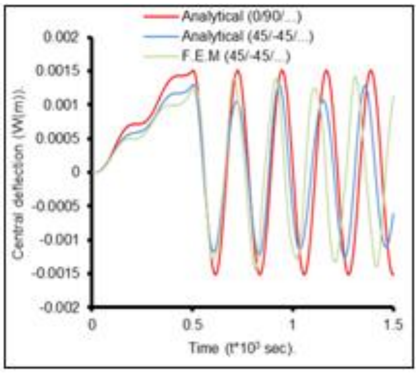

Figure 12: Central Deflection Due to Sinusoidal Ramp Loading (n=4)

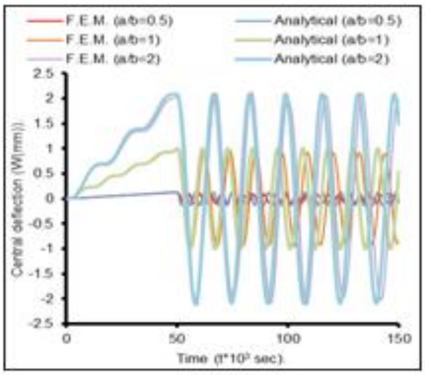

Figure 13: Central Deflection Due to Sinusoidal Ramp Loading for (N=4)

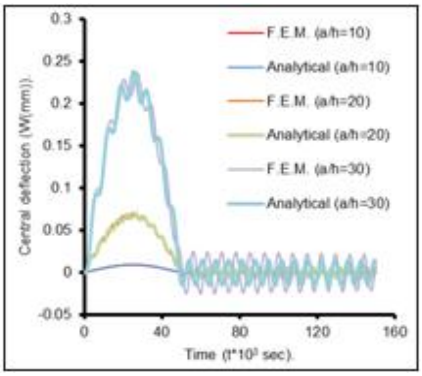

Figure 14: Central Deflection Due to Sinusoidal Sine Loading for (N=4)

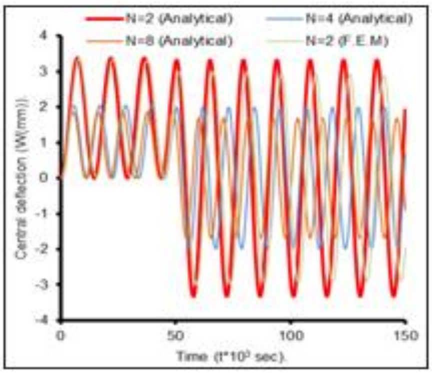

Figure 15: Central Deflection for Sinusoidal Pulse Load, (0/90/…) Plates

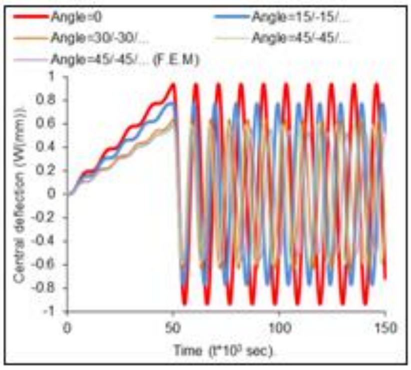

Figure 16: Central Deflection Due to Sinusoidal Ramp Loading for (N=6)

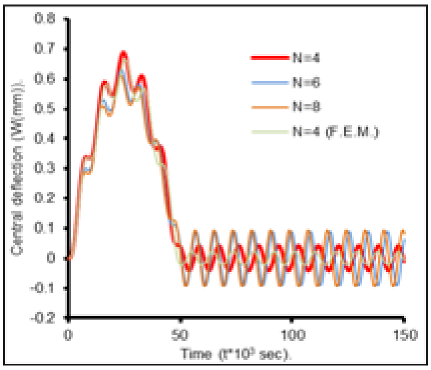

Figure 17: Central Deflection Due to Sinusoidal Sine Loading for (θ=45/-45/…) Plate

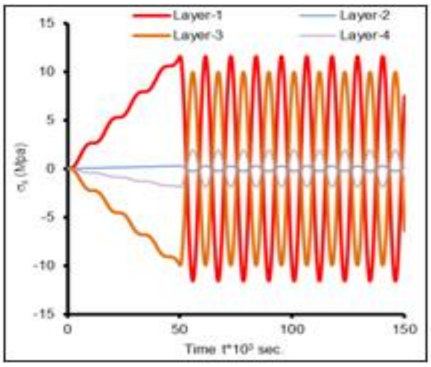

Figure 18: Stress-x in Each Layer Due to Uniform Ramp Loading for (N=4)

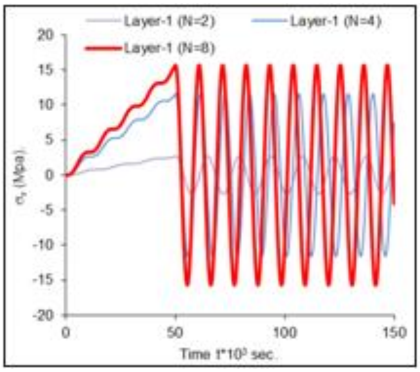

Figure 19: Stress-x in Layer-1 Due to Uniform Ramp Loading

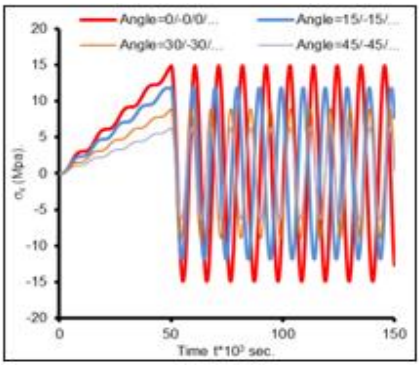

Figure 20: Stress-x at Layer-1 Due to Uniform Ramp Loading for N=4

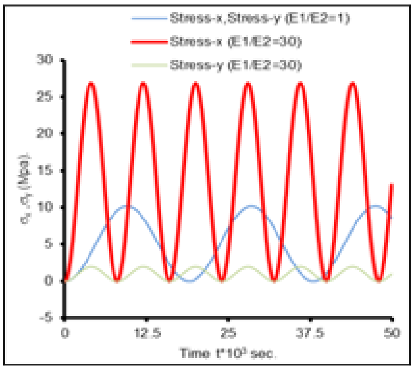

Figure 21: Stress-x and Stress-y in Layer-1 Due to Uniform Pulse Loading for N=4

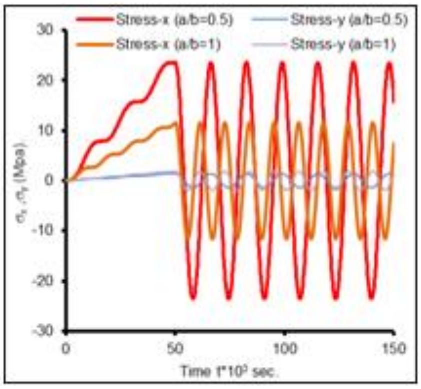

Figure 22: Stress-x and Stress-y in Layer-1 Due to Uniform Ramp Loading for N=4

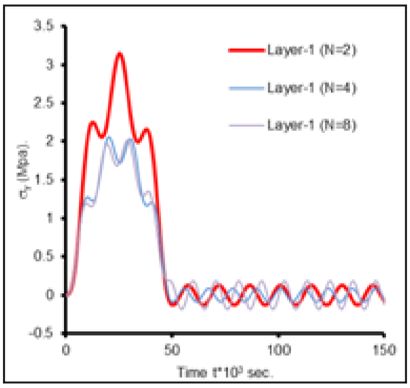

Figure 23: Stress-y in Layer-1 Due to Due to Uniform Sine Loading

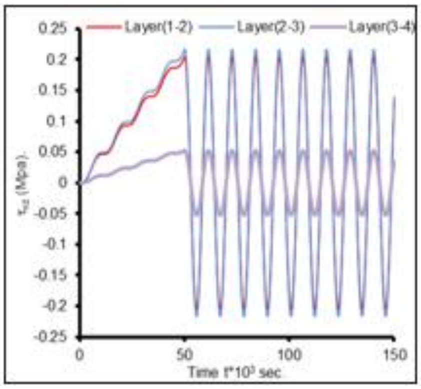

Figure 24: Inter Laminar Shear τ_{xz} Due to Uniform Ramp Loading for Cross-Ply Laminated Plates for N=4

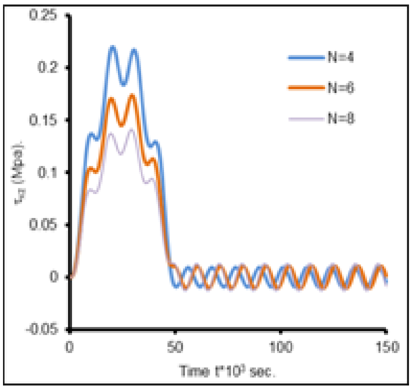

Figure 25: Inter Laminar Shear τ_{xz} in Layers (1-2) Due to Uniform Sine Loading for Cross-Ply Laminated Plates

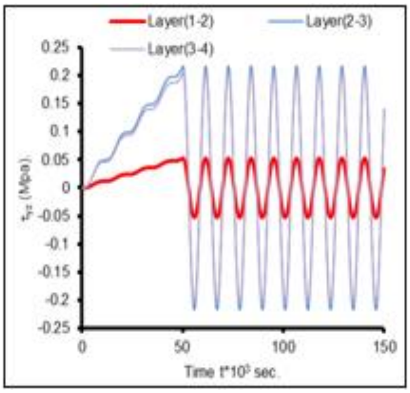

Figure 26: Inter Laminar Shear τ_{yz} Due to Uniform Ramp Load for Cross-Ply Laminated Plates for N=4

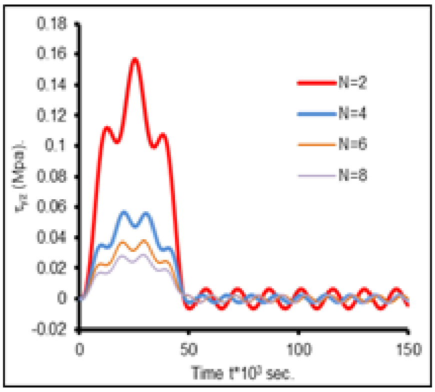

Figure 27: Inter Laminar Shear τ_{yz} in Layers (1-2) Due to Uniform Sine Loading for Cross-Ply Laminated Plates

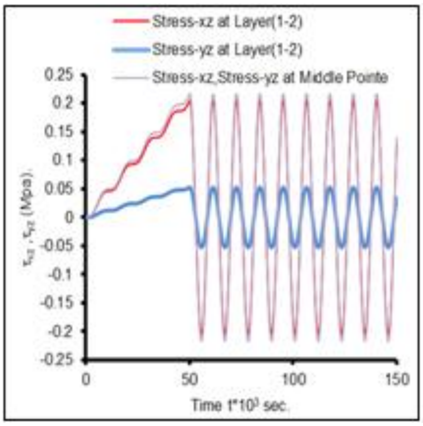

Figure 28: Inter Laminar Shear τ_{xz}, τ_{yz} Due to Uniform Ramp Loading for Cross-Ply Laminated Plates for N=4

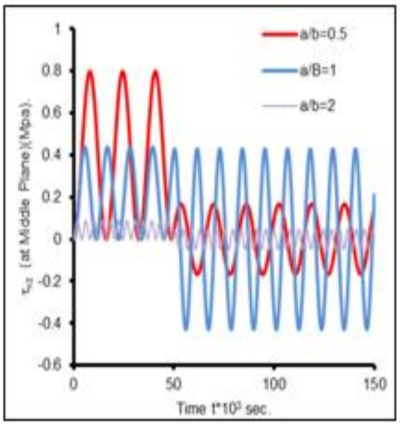

Figure 29: Inter Laminar Shear τ_{xz} Due to Uniform Pulse Loading for Cross-Ply Laminated Plates for N=4

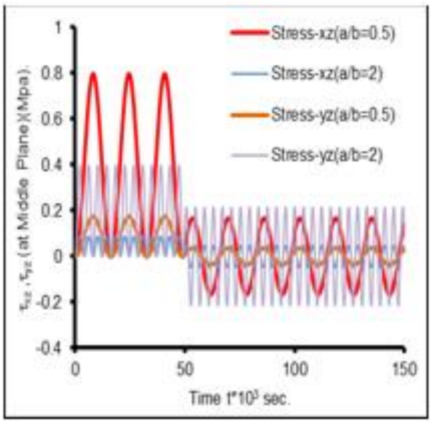

Figure 30: Inter Laminar Shear τ_{xz}, τ_{yz} Due to Uniform Pulse Loading for Cross-Ply Laminated Plates for N=4

CONCLUSIONS

Some concluding observations from the investigation are given below:

- The suggested analytical solution is a powerful tool for solving the differential equation and model analysis method for forced vibration, and, a dynamic stress and inter-laminar shear stress analysis of composite laminated plates.
- The presented work showed that the increasing the numbers of layers for laminated, the angle of fibers, the modules of elasticity E1 more than E2, the aspect ratio, the thickness of laminated decrease the deflection of laminated plates.

- The increasing the aspect ratio or angle of fibers decreases the stress-x, and the increase of number of layer or the E1/E2 ratio increase the stress in direction x, and the increase of the number of layers or the E1/E2 ratio decreases the stress-y.
- The inter laminar shear stresses τxz and τyz are maximum at the middle plane of laminated plates. And, the inter-laminar shear stress τxz at middle plane decreases with increasing the aspect ratio or decreasing the (E1/E2) ratio of laminated plates.

REFERENCES

1. A. A. Khdeir and Reddy, "Dynamic Response of Antisymmetric Angle-Ply Laminated Plates Subjected to Arbitrary Loading" J. Sound and Vibration (1988) 126(3), 437-445.
2. A. K. Nayak, R. A. Shenoi, and S. S. Moy "Transient Response of Initially Stressed Composite Sandwich Plates" Finite Elements **in** Analysis and Design, Vol. 42, pp. 821-836, 2006.
3. Bose, P. and J.N. Reddy (1998) "Analysis of Composite Plates Using Various Plate Theories Part I: Formulation and Analytical Solutions" J. Structural Engineering and Mechanics, Vol. 6, No. 6 , 583-612 .
4. J. N. Reddy, "Dynamic (Transient) Analysis of Laminated Anisotropic Composite-Material Plates" Int. J. Numerical Method in Engineering vol. 19, 237-255 (1983).
5. J. S. Rao "Dynamics of Plates" Narosa Publishing House, (1999).
6. M. Witt and K. Sobczyk "Dynamic Response of Laminated Plates to Random Loading" J. Solids and Structures, Vol. 16, No. 3, (1980).
7. M. Al_Waily "Analysis of Stiffened and Un-Stiffened laminated Plates Subjected to Time Dependant Loading" M.Sc. Thesis Submitted to the Mechanical Engineering Department, College of Engineering, Kufa University, 2005.
8. Pinchas Bar-Yoseph and Jacob Avrashi "Inter-Laminear Stress Analysis for Laminated Plates Containing a Curvilinear Hole" J. Computers and Structures vol. 21, No. 5, p.p. 917-932, (1985).
9. Robert M. Jones "Mechanics of Composite Materials" McGraw-Hill company, (1975).
10. Reddy, J. N. "On The Solutions To Forced Motions of Rectangular Composite Plates" J. Applied Mechanics, ASME ,Vol. 49, (1982).
11. SingiresuS.Rao (1995) "Mechanical Vibrations" Addison-Wesley Publishing Book company.
12. W. L. Yin "Free-Edge Effects in Anisotropic Laminates Under Extension, Bending and Twisting, Part I: a Stress-Function-Based Variational Approach "J. Applied Mechanics, Vol. 61, No. 2, (June 1994).
13. W. L.Yin "Free-Edge Effects In Anisotropic Laminates Under Extension, Bending and Twisting, Part II: Eigen Function Analysis and The Results for Symmetric Laminates" J. Applied Mechanics, Vol. 61, No. 2, (June 1994).
14. ZaferKazanc "Dynamic Response of Composite Sandwich Plates Subjected to Time-Dependent Pressure Pulses" International Journal of Non-Linear Mechanics, Vol. 46, pp. 807-817, 2011.

15. N. VenkataSairam, A. Chandramouli 'Effect of thickness on Interlaminar stresses in clamped FRP angle-ply laminate with a circular cut out' International Journal of Engineering Trends and Technology (IJETT), Vol. 4, No. 7, 2013.

16. KorodyJagannath and K.Jagannath 'Prediction of Inter-laminar Stresses in Composite Laminates using Mixed Finite Element Model' International Journal of Management, IT and Engineering (IJMIE), Vol. 2, No. 5, 2012.

17. C. Fagiano, M. M. Abdalla, C. Kassapoglou, and Z. GurdalInterlaminar Stress Recovery for Three-Dimensional Finite Elements' Composites Science and Technology, Vol. 70, No. 3, 2010.

18. T. Kant and K. Swaminathan Estimation of transverse/interlaminar stresses in laminatedcomposites a selective review and survey of current developments' Composite Structures, Vol. 49, pp.65-75, 2000.